Distinguishing *Gather*-Type and *Numerous*-Type Predicates in the Language of Mathematics

Why Divisiveness-Based Criteria Fail

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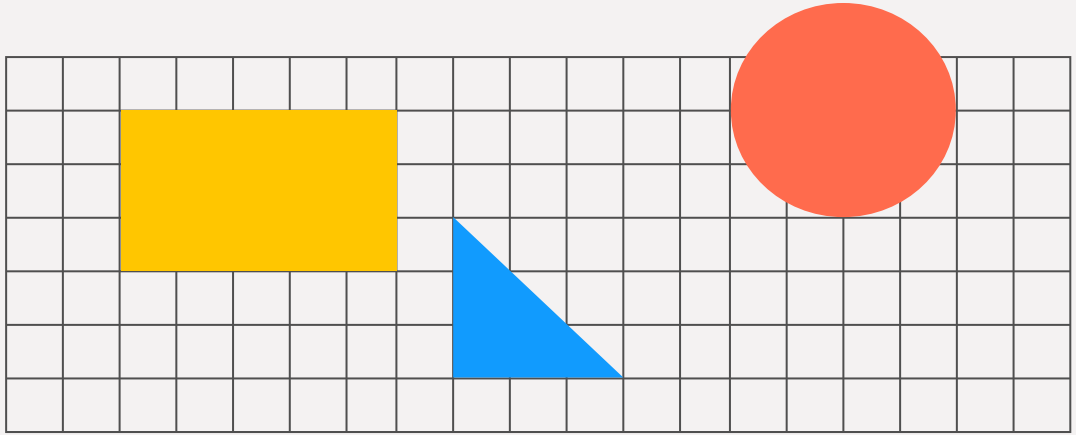


MANCHESTER 1824
The University of Manchester

Distributivity vs Collectivity

(1) A. John and Mary are tall.
B. John is tall and Mary is tall.

(2) A. John and Mary met.
B. # John met and Mary met.

Not all Collective Predicates are the Same

(1) A. The students gathered.
B. The students are numerous.

(2) A. All the students gathered.
B. # All the students are numerous.

Zeno Vendler. Each and every, any and all. Mind, 71(282):145–160, 1962.

David Dowty. Collective predicates, distributive predicates, and all. In Proceedings of the 3rd ESCOL, pages 97–115. Ohio State University Ohio, 1987.

Gather-Type vs. *Numerous*-Type

(1) A. All the students **gathered**.
B. All the puzzle pieces **fit together**.
C. All the students **met**.
D. All the boys are **holding hands**.

***Gather*–Type**

(2) A. # All the students are **numerous**.
B. # All the students are a **large group**.
C. # All the boys are a **good team**.
D. # All the scientists **constitute a majority**.

***Numerous*–Type**

Gather-Type vs. *Numerous*-Type in the Language of Mathematics

LoM: Statements found in mathematics papers and textbooks.

(1)
A. All groups of order 7 are **isomorphic**.
B. All the sets in family P are **equinumerous**.
C. All the lines in the above figure are **parallel**.
D. All the eigenvalues of M are **distinct**.

***Gather*–Type**

(2)
A. # All the elements of X are **coprime**.
B. # All the vectors in V are **orthogonal**.
C. # All the elements of X **differ by at most 1**.
D. # All the points in C are **antipodal**.

***Numerous*–Type**

Ganesalingam, M. (2013). The Language of Mathematics: A Linguistic and Philosophical Investigation. Germany: Springer Berlin Heidelberg.

Why study the Language of Mathematics?

Underexplored in formal semantics:
Test-bed for semantic theories

Foundation for auto-formalisation and math-aware NLP

What Logical Property Distinguishes the two in LoM?

Gather-type

Isomorphic
Equinumerous
Parallel
Distinct

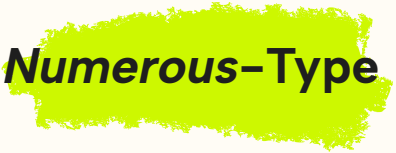
Numerous-type

Coprime
Orthogonal
Differ by at most 1
Antipodal

Suspect 1: Equivalence Relations

(1) A. All groups of order 7 are **isomorphic**.
B. All the sets in family P are **equinumerous**.
C. All the lines in the above figure are **parallel**.
D. All the eigenvalues of M are **distinct**.
E. All the elements of C **have different absolute values.**

(2) A. # All the elements of X are **coprime**.
B. # All the vectors in V are **orthogonal**.
C. # All the elements of X **differ by at most 1**.
D. # All the points in C are **antipodal**.



Arambillete, Santiago, and Philippe de Groote. "On the use of binary relations as collective predicates in natural mathematics." LENLS21-21st International Workshop of Logic and Engineering of Natural Language Semantics. 2025.

Suspect

Equivalence Relations		*Distinct, have different absolute values*

Gather–Type in LoM

Equivalence Relations

Distinct,
Have different absolute values,
Incomparable

Suspect 2: 2-Bounded Divisiveness

If the predicate applies to a collection, it also applies to every non-singleton subcollection.

(1) A. All the boys are similar.
B. # All the boys formed a pyramid.

(2) A. John, Mary and Sam are similar
B. ⇒ John and Mary are similar.

(3) A. John, Mary and Sam formed a pyramid
B. ⇏ John and Mary formed a pyramid.

2–Bounded Divisive

Not 2–Bounded Divisive

Yoad Winter. Flexibility Principles in Boolean Semantics: The Interpretation of Coordination, Plurality, and Scope in Natural Language. The MIT Press, 02 2002.

Gather-Type LoM Predicates are 2-Bounded Divisive

(1) A. The sets $P_1, \ldots, P_n$ are equinumerous.
B. $\Rightarrow$ For $1 \leq i, j \leq n$, the sets P_i and P_j are equinumerous.

(2) A. The groups G_1, G_2, and G_3 are isomorphic
B $\Rightarrow$ The groups G_1 and G_2 are isomorphic.

(3) A. The coeffecients $\lambda_1, \ldots, \lambda_n$ are distinct.
B. $\Rightarrow$ The coefficients λ_1 and λ_2 are distinct.

All 2–Bounded Divisive

Numerous-Type LoM Predicates are trivially 2-Bounded Divisive

In LoM, *numerous*-type predicates apply only to collections of size 2.

(1) A. The vectors v_1 and v_2 are orthogonal.
B. # The vectors $v_1, v_2, \ldots, v_5$ are orthogonal.

Trivially 2–Bounded Divisive

(2) A. The sets A and B differ by at most one element.
B. # The sets A, B, and C differ by at most one element.

(3) A. # The vectors in V are orthogonal.
B. # The sets in the family P differ by at most one element.

Suspects

Equivalence Relations		*Distinct, have the same absolute value*
2-Bounded Divisive		*Numerous*-type do not apply to collections of size more than 2

2-Bounded Divisiveness

***Gather*-type in LoM**
Isomorphic
Equinumerous
Parallel
Distinct

***Numerous*-type in LoM**
Coprime
Orthogonal
Differ by at most 1
Antipodal

2-Bounded Divisiveness Undergenerates in NL

(1) A. All the boys in the circle held hands.

B. All the pieces of the puzzle fit together.

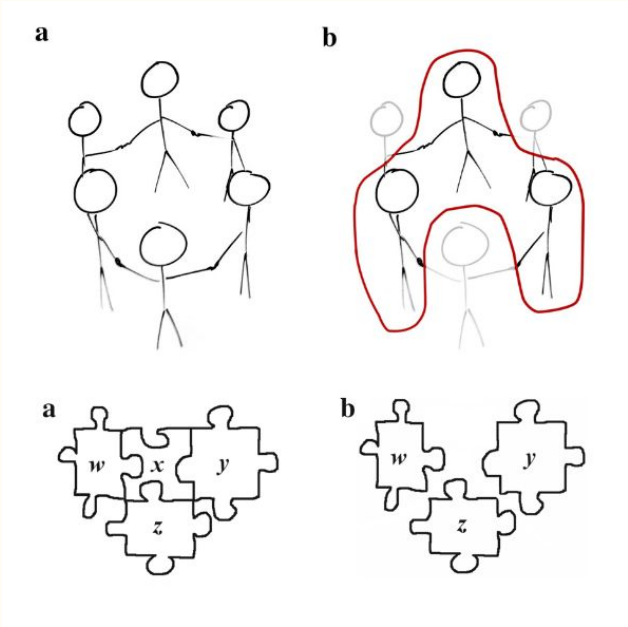
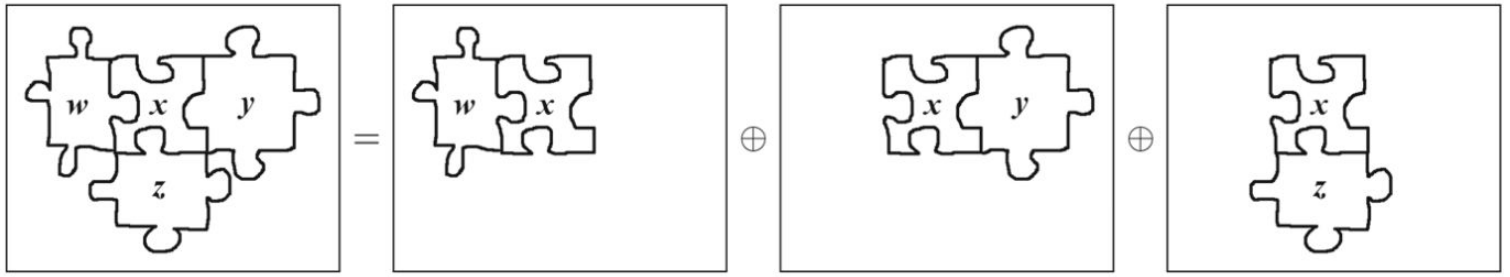


Figure from Jeremy Kuhn. Gather/numerous as a mass/count opposition. Natural Language Semantics, 28(3):225–253, 2020.

Suspect 3: Stratified Reference (SR)

SR: If the predicate applies to a collection, the collection can be divided into, possibly overlapping, subcollections of sufficiently small size, such that the predicate applies to each of the subcollections.



Lucas Champollion. Stratified reference: the common core of distributivity, aspect, and measurement. Theoretical Linguistics, 41(3-4):109–149, 2015

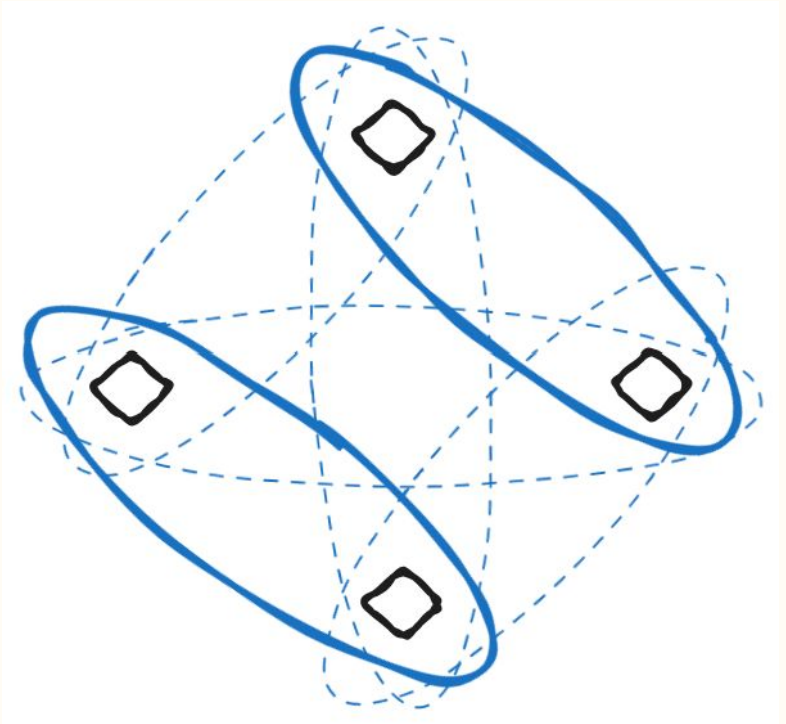
Figure from Jeremy Kuhn. Gather/numerous as a mass/count opposition. Natural Language Semantics, 28(3):225–253, 2020.

2-Bounded Divisiveness implies Stratified Reference

The predicate applies to a collection

It applies to every non-singleton subcollection

The collection can be divided into, overlapping subcollections of size 2 such that that the predicate applies to each of the subcollections.



Suspects

Equivalence Relations		*Distinct, have the same absolute value*
2-Bounded Divisive		*Numerous*-type predicates do not apply to collections of size more than 2
Stratified Reference		2-bounded divisiveness implies stratified reference

Stratified Reference Overgenerates in NL

(1) A. # All my social media followers are fewer than ten in number.

B. # All the trees are denser in the middle of the forest.

Any collection can be divided into subcollections of size less than 10

Jeremy Kuhn. Gather/numerous as a mass/count opposition. Natural Language Semantics, 28(3):225–253, 2020.

Suspect 4: Grounded Stratified Reference (GSR)

Accidental Satisfaction!

GSR: If the predicate applies to a collection X, the collection can be divided into, possibly overlapping, subcollections of sufficiently small size, such that the predicate applies to each of the subcollections,
but there exists an accessible world in which X cannot be so divided.

Fewer than ten in number does not have GSR

Jeremy Kuhn. Gather/numerous as a mass/count opposition. Natural Language Semantics, 28(3):225–253, 2020.

GSR implies Worlds with Different Mathematics

For every collection X, there is an accessible world such that X cannot be divided into, possibly overlapping, subcollections Ys of sufficiently small size (ϵ_0), such that the predicate P applies to each of the Ys. For some $\epsilon_0 > 1$,

$$\forall A\,[\,\cup A = X \rightarrow \exists Y \in A\,[|Y| \leq \epsilon_0 \rightarrow \neg P(Y)]]$$

The natural numbers and the integers are equinumerous.

$$\cup\{\{N,Z\}\} = \{N,Z\} \rightarrow [|\{N,Z\}| \leq \epsilon_0 \rightarrow \neg E(\{N,Z\})]$$

1. $\cup\{\{N,Z\}\} \neq \{N,Z\}$
2. $|\{N,Z\}| > \epsilon_0 \geq 2$
3. $\neg E(\{N,Z\})$

Suspects

Equivalence Relations		*Distinct, have the same absolute value*
2-Bounded Divisive		*Numerous*-type predicates do not apply to collections of size more than 2
Stratified Reference		2-bounded divisiveness implies stratified reference
Grounded Stratified Reference		Accessible worlds have non-standard math

Other Candidates?

Atom vs Set Predicates

- *Gather :* (et)t
- *Numerous :* et

- *Parallel* : (et)t
- *Orthogonal* : et

Puzzling for LoM!

Winter, Yoad. Flexibility principles in Boolean semantics: The interpretation of coordination, plurality, and scope in natural language. MIT press, 2002.

- *Gather*-type: Activities, Accomplishments.

- *Numerous*-type predicates: States, Achievements.

Zeno Vendler. Verbs and times. The Philosophical Review, 66(2):143–160, 1957.

Alison Taub. Collective predicates, aktionsarten and all. In Emmon Bach, Angelika Kratzer, and Barbara H. Partee, editors, Papers on quantification, pages 337–366. University of Massachusetts, Amherst, 1989

C. Brisson. Plurals, all, and the nonuniformity of collective predication. Linguistics and Philosophy, 26(2):129–184, 2003.

Observation: Different Behaviour of *Numerous*-type Predicates

In natural language:

A. The boys are a good team.
B. The students formed a pyramid.
C. The left-leaning MPs constitute a majority.

In LoM:

A. # The elements of X differ by 1
B. # The vectors in V are orthogonal.
C. # The sets in the family P differ by at most one element.

Counterexample: *Coprime*?

(1) A. 2 and 3 are coprime.
B. # 2 is coprime.

Collective

(2) # All the elements of X are coprime.

***Gather*–Type**

(3) A. The integers 6, 10, 15 are coprime.
B. ⇏ The integers 6 and 10 are coprime.

Not 2–Bounded Divisive?

Two "*Coprime*"s

- If gcd(a, b) = 1, then we say a and b are coprime.
- A set S is coprime if gcd(S) = 1.

(1) A. 2 and 3 are coprime_p.
B. The finite set {6, 10, 15} is coprime_s.

(2) A. The sets X, Y ⊆ Z are coprime_s .
B. The set X is coprime_s and the set Y is coprime_s .

(2) A. The integers 6, 10, 15 are coprime ⇔ The set {6, 10, 15} is coprime_s.
C. ⇏ The integers 6 and 10 are coprime_s.

University of Texas at Austin Department of Mathematics. Introduction to number theory: Course notes, 2025. Accessed: 2025-11-04.

Observation: Dual Reference

(1) A. 6, 10, and 15 are coprime.
B. The finite set {6, 10, 15} is coprime.

(2) A. The integers are countable.
B. The set of integers is countable.

(3) A. The integers and the rationals are equinumerous.
B. The set of integers and the set of rationals are equinumerous.

(4) A. The vectors v_1 , v_2 and v_3 are linearly dependent.
B. The set $\{v_1 , v_2 , v_3 \}$ is linearly dependent.